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5-4 day 2 The FUNDamental Theorem of Calculus

Learning Objectives:

I understand the connection between integral and differential calculus.

I can evaluate an integral using the Fundamental Theorem of Calculus Part 2.

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$f(x) = 2x$

$g(x) = \int_0^x f(t) dt$

x	g(x)
0	0
1	1
2	4
3	9
4	16
5	25

$\int_0^1 f(t) dt$
 $\frac{1}{2} \cdot 2 \cdot 4$
 $\frac{1}{2} \cdot 3 \cdot 6$
 $\frac{1}{2} \cdot 4 \cdot 8$

$g(x) = x^2$

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$f(t) = 2t$

$g(x) = x^2$

Antiderivative

Derivative

$g(x)$ is the antiderivative of $f(t)$

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$y = \frac{1}{2} x^2$

$\int_0^4 \frac{1}{2} x^2 dx$ ← derivative

$\frac{1}{6} x^3$ ← finds the area that accumulates

$\frac{1}{6} (4)^3 = \frac{64}{6} = \frac{32}{3} \approx 10.667$

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$y = \frac{1}{2} x^2$

$\int_2^5 \frac{1}{2} x^2 dx = \int_0^5 \frac{1}{2} x^2 dx - \int_0^2 \frac{1}{2} x^2 dx$

$\frac{1}{6} x^3$

$\frac{1}{6} (5)^3 - \frac{1}{6} (2)^3$

19.5 units²

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The FUNdamental Theorem of Calculus Part 2

If $f(x)$ is continuous at every point in $[a, b]$ and if $F(x)$ is the antiderivative of $f(x)$ on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a)$$

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Ex1. Evaluate. Check your answer on the graphing calculator

1.) $\int_{-2}^5 (x^2 + 3x + 1) dx$

$$\begin{aligned} & \left(\frac{1}{3}x^3 + \frac{3}{2}x^2 + x \right) \Big|_{-2}^5 \\ & \left(\frac{1}{3} \cdot 5^3 + \frac{3}{2} \cdot 5^2 + 5 \right) - \left(\frac{1}{3}(-2)^3 + \frac{3}{2}(-2)^2 + (-2) \right) \\ & = \frac{125}{3} + \frac{75}{2} + 5 + \frac{8}{3} - \frac{12}{2} + 2 \\ & = 82.533 \quad \frac{133}{3} + \frac{63}{2} + 7 \\ & \quad \frac{266}{6} + \frac{189}{6} + \frac{42}{6} \end{aligned}$$

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2.) $\int_1^4 (e^{2x}) dx$

$$\frac{1}{2} e^{2x} \Big|_1^4$$

$$\frac{1}{2} e^8 - \frac{1}{2} e^2$$

$$\frac{e^{2x}}{2}$$

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3.) $\int_0^{\pi} \sin x dx$

$$\begin{aligned} & -\cos x \Big|_0^{\pi} \\ & -\cos \pi - (-\cos 0) \\ & -(-1) - (-1) \\ & 1 + 1 \\ & 2 \end{aligned}$$

$$\cos x$$

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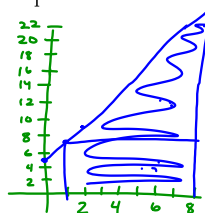
4.) $\int_{-1}^3 \frac{dx}{x+5}$

$$\ln(x+5) \Big|_{-1}^3$$

$$\ln(8) - \ln(4) = \ln\left(\frac{8}{4}\right) = \ln(2)$$

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4.) $\int_1^8 (2x+5) dx$



$$\begin{aligned} & = (x^2 + 5x) \Big|_1^8 \\ & = 8^2 + 40 - 1^2 - 5 \\ & = 98 \end{aligned}$$

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Ex2. Find the area between the curve

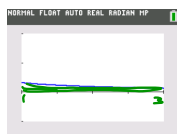
$$f(x) = \frac{1}{2x+1} \text{ and the } x\text{-axis}$$

bounded by $1 \leq x \leq 3$

$$\int_1^3 \left(\frac{1}{2x+1}\right) dx$$

$$= \frac{1}{2} \ln(2x+1) \Big|_1^3$$

$$= \frac{1}{2} \ln(7) - \frac{1}{2} \ln(3) = \frac{1}{2} \ln\left(\frac{7}{3}\right)$$



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Ex3. Find the area between the curve

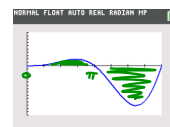
$$f(x) = x^2 \sin x \text{ and the } x\text{-axis}$$

bounded by $0 \leq x \leq 2\pi$

$$\int_0^\pi x^2 \sin x dx = 5.8696$$

$$\int_\pi^{2\pi} x^2 \sin x dx = 45.3478$$

$$= 51.218 \int_0^{2\pi} |x^2 \sin x| dx$$



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Homework

pg 302 # 27, 29, 30, 32-35, 38,
39, 42, 43, 45-50

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